Assignment 1

Last Name: Vernon; **First Name:** Kori; **NetID:** ksv244; **Section:** TWTH 10:00-12:50

(1) (10 points) Show that if a, b, c and d are integers with a and c nonzero such that $a|b$ and $c|d$, then $ac|bd$.

- If $a|b$, then there must be some integer x such that $ax = b$. There should also be some integer y such that $cy = d$. If we substitute values, we can see that $ac|bd = ac|axcy = ac|ac(xy)$. Because ac divides itself, then $ac|bd$ holds true $\Box$.

(2) (10 points) Show that if a, b and $c \neq 0$ are integers, then $a|b$ if and only if $ac|bc$.

"$\Rightarrow$" If $a|b$, then there must be some integer x such that $ax = b$. If c is on both sides, it is as if it is a constant. If we substitute values, we can see that $ac|(ax)c = ac|ac(x)$. Because $ac|ac$, this holds true.

"$\Leftarrow$" If $ac|bc$, then there must be some integer y such that $ac(y) = bc$ which is equal to $ay = b$. If we substitute values, we can see that $a|ay$, because $a|a$, therefore this statement holds true $\Box$.

(3) (5 points) Show that the sum of two even (i) or of two odd integers is even (ii), while the sum of an odd and an even integer is odd (iii).

(i) The sum of two even numbers is even because suppose we have two integers x, and y. We have one even number $2x$, and another even number $2y$, and we add them together $2x + 2y = 2(x + y)$. That number is divisible by 2. $2|2(x + y)\checkmark$.

(ii) The sum of two odd integers is always even because suppose we have two integers u, and v. We have one odd number $2u + 1$, and another odd number $2v + 1$, and we add them together $2v+2u+2 = 2(u+v+1)$. This number is divisible by 2. $2|2(u+v+1)\checkmark$.

(iii) The sum of one odd integer and one even integer is always odd, because suppose we have two integers n and k. We have one odd number $2n + 1$, and one even number $2k$. If we add the even and odd number together, we get $2n + 2k + 1 = 2(k + n) + 1$. This number is **NOT** divisible by 2. $2 \nmid 2(k + n) + 1\checkmark$.

(4) (5 points) Show that the product of two integers of the form $6k + 5$ is of the form $6k + 1$ where k is some integer.

- Assume there is an integer $x \in \mathbb{Z}$ and $y \in \mathbb{Z}$, where $x = 6b + 5$, and $y = 6c + 5$, where b and c are arbitrary integers.

  $x \cdot y = (6b + 5) \cdot (6a + 5) = 36ba + 30a + 30b + 25 = 6(6ab + 5a + 5b + 4) + 1 = 6k + 1\Box$

(5) (8 points) Construct the truth table of $[(p \vee q) \wedge r] \rightarrow (p \wedge \neg q)$.

p	q	r	$(p \vee q) \wedge r$	$(p \wedge \neg q)$	$(p \vee q) \wedge r \rightarrow (p \wedge \neg q)$
T	T	T	T	F	F
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	T
F	T	T	T	F	F
F	F	T	F	F	T
F	T	F	F	F	T
F	F	F	F	F	T

(6) Four married couples have bought 8 seats in the same row for a concert. In how many ways can they be seated:

(a) With no restrictions?

- 8! Because there are 8 different choices for the first seat, so on and so forth.

(b) If each couple is to sit together?

- $4! \cdot (2!)^4$ because there are 4 potential spots that the couples could sit if they were to be together, those spots can be arranged 2! ways, and there are 4 different couples.

(c) If all of the men sit together to the right of all of the women.

- $4! \cdot 4!$ because if the men sit to the right of the women, then they could be arranged 4! different ways. The women can also be arranged 4! different ways.

Note: *Assuming that they are heterosexual couples...*

(7) (6 points) Find the cardinality of the following sets.

(a) $2^{2^{1,2,3}}$

- The cardinality of $2^{\{1,2,3\}} = 8$. $2^8 = 256$

(b) $\{x \in \mathbb{Z} : x \in \emptyset\}$

- The cardinality of this set is 0, because the cardinality of the emptyset is 0. Since the result of this set is an emptyset, the cardinality is 0.

(c) $\{x \in 2^{\{1,2,3,4\}} : |x| = 1\}$

- The cardinality of the powerset when it is equal to one.
- The result of the sets with cardinality one will be $\{1\}, \{2\}, \{3\}, \{4\}$, in which the cardinality would be 4.